Review 3.13–3.18 KEY

Name_____ Date _____ Period _____

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

Find the component form and magnitude of the indicated vector.

1) Given that P = (–2, 7) and Q = (–4, –2), find the component form and magnitude of the vector $3\overrightarrow{PQ}$ 1) _____

$3\overrightarrow{PQ} = 3\langle(-4-(-2)), (-2,-7)\rangle$

$= 3\langle -2, -9\rangle = \langle -6, -27\rangle$

$|3\overrightarrow{PQ}| = \sqrt{(-6)^2 + (-27)^2}$

$= \sqrt{765}$

or $3\sqrt{85}$

Find the component form of the indicated vector.

2) Let $\mathbf{u} = \langle -7, 1\rangle$, $\mathbf{v} = \langle 8, 3\rangle$. Find $\mathbf{v} - \mathbf{u}$. 2) _____

$V - U = \langle 8-(-7), 3-1\rangle = \langle 15, 27\rangle$

3) Let $\mathbf{u} = \langle 8, -4\rangle$, $\mathbf{v} = \langle -1, 2\rangle$. Find $2\mathbf{u} - \mathbf{v}$. 3) _____

$2u = \langle 16, -8\rangle - \langle -1, 2\rangle = \langle 17, -10\rangle$

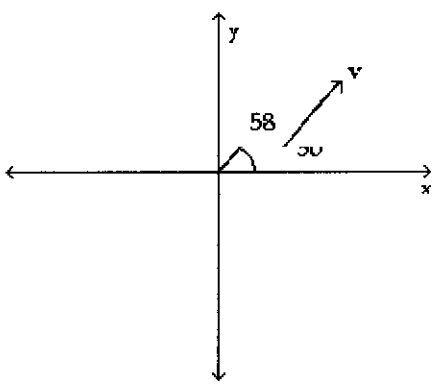
Find the unit vector in the direction of the given vector. Write your answer in the indicated form.

4) Let $\mathbf{u} = \langle 3, 4\rangle$. Find the unit vector in the direction of $\mathbf{u}$, and write your answer in component form. 4) _____

$\text{Unit vector} = \frac{u}{|u|} = \frac{\langle 3, 4\rangle}{5} = \left\langle \frac{3}{5}, \frac{4}{5} \right\rangle$

Find the component form of the vector v.

5) 5) _____



$V = \langle V_1, V_2\rangle$

$V_1 = 58\cos 50° = 37.28$

$V_2 = 58\sin 50° = 44.43$

$V = \langle 37.28, 44.43\rangle$

Find the magnitude and direction angle for the following vector. Give the direction angle as an angle in [0°, 360°) rounded to the nearest tenth.

6) ⟨–2, –7⟩ → Quad. 3 6) _____

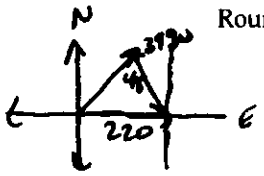
$|\langle -2, -7\rangle| = \sqrt{(-2)^2 + (-7)^2} = \sqrt{53}$

$\theta = \tan^{-1}\left(\frac{-7}{-2}\right) = 74° + 180° = 254°$

Solve the problem.

7) An airplane flies on a compass heading of 90.0° at 220 mph. The wind affecting the plane is blowing ~~from~~ 321° at 48.0 mph. What is the true course and ground speed of the airplane? Round results to an appropriate number of significant digits. 7) _____

with bearing of



$A = \langle 220\cos 0°, 220\sin 0° \rangle = \langle 220, 0 \rangle$

$w = \langle 48\cos 129°, 48\sin 129° \rangle = \langle -30.21, 37.37 \rangle$

$A + w = \langle 189.79, 37.37 \rangle \quad \theta = \tan^{-1}\left(\frac{37.3}{189.79}\right) = 11.11$

$90 - 11.11 = 78.88$

ground speed: $\sqrt{(189.79)^2 + (37.3)^2} =$ 193.42 mph.

Find a · b.

8) $\mathbf{a} = \langle 4, 1 \rangle$, $\mathbf{b} = \langle 3, -2 \rangle$ 8) _____

$a \cdot b = (4)(3) + (1)(-2) = 12 - 2 = 10$

Use the dot product to find $|\mathbf{v}|$.

9) $\mathbf{v} = \langle -6, 1 \rangle$ 9) _____

$\sqrt{v \cdot v} = \sqrt{(-6)(-6) + (1)(1)} = \sqrt{36 + 1} = \sqrt{37}$

Find the angle between the given vectors to the nearest tenth of a degree.

10) $\mathbf{u} = \langle 6, 3 \rangle$, $\mathbf{v} = \langle -5, -3 \rangle$ 10) _____

$\theta = \cos^{-1}\left(\frac{u \cdot v}{|u||v|}\right)$

$\theta = \cos^{-1}\left(\frac{(6)(-5) + (3)(-3)}{\sqrt{45} \cdot \sqrt{34}}\right) = \frac{-39}{\sqrt{1530}} = 175.6°$

11) $\mathbf{u} = \mathbf{i} + \sqrt{7}\mathbf{j}$, $\mathbf{v} = -\mathbf{i} - 4\mathbf{j}$ 11) _____

$\theta = \cos^{-1}\left(\frac{(1)(-1) + (\sqrt{7})(-4)}{\sqrt{8} \cdot \sqrt{17}}\right) = \frac{-1 - 4\sqrt{7}}{\sqrt{136}} = 173.3°$

Determine whether the vectors u and v are parallel, orthogonal, or neither.

12) $\mathbf{u} = \langle 1, 6 \rangle$, $\mathbf{v} = \langle 4, 24 \rangle$ 12) _____

$\frac{6}{4} = \frac{24}{4}$ Parallel

13) $\mathbf{u} = \langle 6, 5 \rangle$, $\mathbf{v} = \langle -5, 2 \rangle$ 13) _____

$(6)(-5) + (5)(2) = -30 + 10 = -20$ Neither

$\frac{5}{6} \neq \frac{2}{-5}$

Eliminate the parameter.

14) $x = t + 4$, $y = t^2$ 14) _____

$t = x - 4 \qquad y = (x-4)^2$

$y = x^2 - 8x + 16$

Find the rectangular coordinates of the point with the given polar coordinates.

15) $(-4, -\pi/3)$ 15) _____

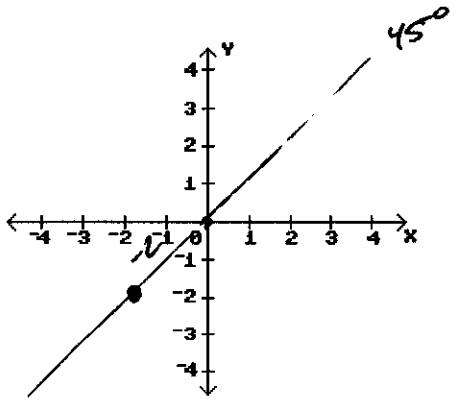
$x = r\cos\theta \qquad x = -4\cos(-\pi/3) = -4\left(\frac{1}{2}\right) = -2$

$y = r\sin\theta \qquad y = -4\sin(-\pi/3) = -4\left(-\frac{\sqrt{3}}{2}\right) = 2\sqrt{3}$

$(-2, 2\sqrt{3})$

Plot the point with the given polar coordinates.

16) $\left(-2, \frac{\pi}{4}\right)$



16) _____

Determine two pairs of polar coordinates for the point with $0° \le \theta < 360°$.

17) $(2\sqrt{3}, 6)$ $r = \sqrt{(2\sqrt{3})^2 + (6)^2} = 4\sqrt{3}$

$\theta = \tan^{-1}\left(\frac{6}{2\sqrt{3}}\right) = 60°$

(r, θ)

$(4\sqrt{3}, 60°)$

$(-4\sqrt{3}, 240°)$

17) _____

Find an equivalent equation in rectangular coordinates.

18) $r = 10 \sin\theta$

$r \cdot r = 10 r \sin\theta$

$r^2 = 10 r \sin\theta$

$x^2 + y^2 = 10y$ or

$x^2 + y^2 - 10y = 0$

18) _____

19) $r = \cos\theta$

$r \cdot r = r\cos\theta$

$r^2 = r\cos\theta$

$x^2 + y^2 = x$ or $x^2 - x + y^2 = 0$

19) _____

Express the complex number in trigonometric form.

20) 2 $r = 2$ $\theta = 0°$

$2(\cos 0° + i \sin 0°)$

20) _____

Write the complex number in the form a + bi.

21) $8(\cos 30° + i \sin 30°)$

$8 \cdot \frac{\sqrt{3}}{2} + 8 \cdot \frac{1}{2} i = 4\sqrt{3} + 4i$

21) _____

Find the product or quotient, as indicated. Leave your answer in trigonometric form.

22) Find the product of z_1 and z_2.

$z_1 = 7(\cos 70° + i \sin 70°)$, $z_2 = 5(\cos 155° + i \sin 155°)$

$z_1 \cdot z_2 = 7 \cdot 5 (\cos(70° + 155°) + i \sin(70° + 155°))$

$= 35(\cos 225° + i \sin 225°)$

22) _____

23) Find the quotient.
$$\frac{7(\cos 40° + i \sin 40°)}{4(\cos 295° + i \sin 295°)}$$

$\frac{z_1}{z_2} = \frac{7}{4}(\cos(40°-295°) + i\sin(40°-295°))$

$= \frac{7}{4}(\cos(-255°) + i\sin(-255°))$ 23) _____

Find the product or quotient. Write the answer in standard form.

24) $(3 - 7i)(8 + 3i)$ 24) _____

$24 + 9i - 56i - 21i^2$

$24 - 47i + 21 = 45 - 47i$

25) $\frac{8 + 6i}{4 - 2i} \cdot \frac{4+2i}{4+2i} = \frac{32 + 16i + 24i + 12i^2}{4^2+2^2} = \frac{32 + 40i - 12}{16+4} = \frac{20+40i}{20}$ 25) _____

$= 1 + 2i$

Use De Moivre's Theorem to find the indicated power of the complex number. Write your answer in standard form a + bi.

26) $(2 - 2i)^5$ $r = \sqrt{(2)^2+(-2)^2} = 2\sqrt{2}$ $\theta = \tan^{-1}(-1) = \frac{7\pi}{4}$ 26) _____

$(2\sqrt{2})^5\left[\cos\left(5 \cdot \frac{7\pi}{4}\right) + i\sin\left(5 \cdot \frac{7\pi}{4}\right)\right]$

$(2\sqrt{2})^5\left(\cos\frac{35\pi}{4} + i\sin\frac{35\pi}{4}\right) = -128 + 128i$

Find the indicated roots. Write the answer in a + bi form.

27) Cube roots of 8 $r = 8, \theta = 0$ 27) _____

$z_1 = \sqrt[3]{8}(\cos 0 + i\sin 0) = 2 + 0i$

$z_2 = \sqrt[3]{8}(\cos 120° + i\sin 120°) = -1 + \sqrt{3}i$

$z_3 = \sqrt[3]{8}(\cos 240° + i\sin 240°) = -1 - \sqrt{3}i$

Find the indicated roots. Write the answer in trigonometric form.

28) Cube roots of $8(\cos 243° + i \sin 243°)$ $\frac{243}{3} = 81$ $\frac{360}{3} = 120$ 28) _____

$81 + 120 = 201°$

$201° + 120 = 321°$

$z_1 = 2(\cos 81° + i\sin 81°)$

$z_2 = 2(\cos 201° + i\sin 201°)$

$z_3 = 2(\cos 321° + i\sin 321°)$

Provide an appropriate response.

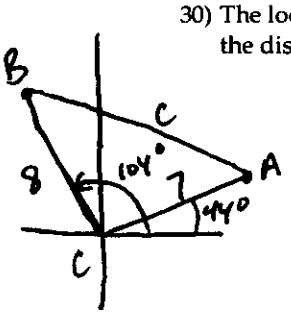
29) Determine whether each statement below is true or false. If either statement is false, provide a counterexample. 29) _____

- If two vectors are perpendicular, then their dot product must be zero. True
- If two vectors are orthogonal, then they must also be perpendicular. False

If one is the zero vector then they will be orthogonal but not perpendicular.

Solve the problem.

30) The locations, given in polar coordinates, of two ships are (7 mi, 44°) and (8 mi, 104°). Find the distance between the two ships. 30) _____



$\angle C = 104° - 44°$

$\angle C = 60°$

Law of Cosines

$c^2 = 8^2 + 7^2 - 2(8)(7)\cos 60°$

$c^2 = 57$

$c \approx 7.55$ miles